A MODEL MATHEMATICS PROGRAM FOR THE ACADEMICALLY DEPRIVED CHILD

A Field Report

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by

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CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

The central administration of the Webster City Community School District has expressed a need for a program to meet the needs of the academically deprived child in the area of mathematics. In response to this expressed need a special program was to be developed.

It is felt that through this new program the academically deprived child will no longer need to be forced into concepts they do not understand or kept at a level they have already achieved through a program structured around one text.

This new model program includes not only students who are academically deprived because of the lack of skills for their ability, but also includes enrichment for those students who have been academically deprived from enrichment in accordance with their advanced ability.

I. THE PROBLEM

Statement of the problem. It was the purpose of this study to develop a model program in the area of mathematics for extending academic help to the academically deprived student in the elementary grades, and adapting the model to the Webster City Community School District.

<u>Importance of the study</u>. The necessity of this study was in its importance of the central administration of the Webster City Community School District. It was essential that a mathematics program for the academically deprived child be developed if this district was to meet its responsibilities.

The program was limited to elementary. It is here that the fundamentals for future development of mathematical principles are needed. The purpose of the program was to give opportunity to more capable students to extend their skills. For students who lack understanding of basic skills it offered remedial work in the skill areas.

<u>Limitation</u>. The model program was limited to elementary mathematics, and applied to the Webster City Community School District in Webster City, Iowa.

<u>Principles</u>. The model mathematics program for the academically deprived child was developed on the following psychological principles[1] of learning:

1. Each individual was unique and therefore did not all learn in the same manner.
2. Psychologically speaking there was no such thing as a grade level. It was a place, not a state of being.

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[1]Psychological principles as taught by Dr. Weakly of Drake University, Des Moines, Iowa. Education Courses #220 "Psychological Foundations of Basic Skills," and #281 "Social Psychology of Education." 1969.

3. Learning proceeded best and was more permanent when the purposes were understood by the learner.

4. Learning involved the building of relationships.

5. The learner recognized he was learning for bases for further learning.

6. Curriculum was the vehicle around which the learning took place...not the body per say.

7. A part of the responsibility for the learner did rest on the shoulder of the learner. The learner experienced a growing acceptance of responsibility.

8. School activities had a social implication.

Criteria. The criteria of the mathematics program was:

1. Understanding the number system was primary.

2. Secondary was operating the number system.

3. The instructor found where the child was in the developmental stage and worked from there.

4. Goals were set in the functional needs, and then were adjusted to the learner.

5. Goals were set in understanding of how to solve problems and why, and then were adjusted to the needs of the learner.

Procedure. This model mathematics program for the academically deprived child was developed as a flexible program that gives aid and consideration to the individual child who finds it difficult to meet his needs in mathematics.

The program is to identify the learner in need, to determine his needs, and to adjust materials to fill his need. The instructor evaluates the learner's progress for further learning needs.

This program was done on an experimental basis within an elementary building in the Webster City Community School District.

The procedure was to develop a simple teacher rating form for identifying the learners, stating their needs, and evaluating their progress. The instruction and adjustment of material to the learner is flexible and is dependent upon the instructor. In order to keep the program flexible it was necessary to develop a teacher's guide.

Guide. A general teaching of arithmetic guide was written as an operational part of the model program because this program was written to be used with any materials that can be adjusted to the needs of the learner.

II. DEFINITION OF TERM USED

Academically deprived child. Academically deprived child refers to any child who is not achieving to his ability for any reason.

In order to develop a mathematics program for the academically deprived child an awareness of the individual child was needed. An awareness of the individual child and the relationship between the individual and learning is discussed in the related literature.

In this chapter is the statement of the problem and the definition of the term academically deprived child. The next chapter is the review of the literature on the individual child and topics of curriculum. The following chapters will include the procedures and data on the mathematics program. The next to the last chapter is the mathematics guide written to be used with most existing mathematics programs.

CHAPTER II

REVIEW OF THE LITERATURE

Topics that were closely related to the development of a program for the individual child were searched, and brief summary of the writings on the topics of curriculum, individual child development, and individual child's ability follow.

Kingsley and Garry discussed the principle of learning that indicates an individual is unique, and that there is a relationship between his uniqueness and the development of concepts in the acquisition of educational and social needs. An "individual's variables must be considered simultaneously by the teacher in planning learning activities for individual children."[1] An instructing professional has to be able to understand the relationships between location and isolation steps that a child utilized in developing abstract concepts, and "to develop, refine, and elaborate on concepts of the relationship to perceive analytically."[2]

Buck and Zarfoss in reviewing the principles of learning concluded that the individual child is unique and

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[1]Howard L. Kingsley and Ralph Garry, The Nature and Conditions of Learning (New Jersey: Prentice-Hall, Inc., 2nd Edition, 1957), p. 135.

[2]Ibid., p. 406.

cannot be treated the same as others. They reasoned that if individuals were unique then "no two classes regular or special can be exactly alike."[1]

Keppel discussed the need to consider the variables that affect a curriculum, and stressed that we must link these variables together into a system of change. He felt that equipment and curriculum, teaching methods and school organization should be linked together. These factors should be linked to the preparation of teachers and to the measurement of the results in what students learn.[2]

There are many variables affecting the individual child and such variables as the psychological and cultural differences should be considered in the planning and execution of the curriculum.[3]

In teaching individual children Meyen stated that he felt it was the school's responsibility "for structuring a differentiated curriculum,"[4] to take care of the needs of the

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[1]Pearl S. Buck and Gweneth Zarfoss, The Gifts They Bring Our Debt to the Mentally Retarded (New York: The John Day Co., 1965), p. 67.

[2]Frances Keppel, The Necessary Revolution in American Education (New York: Harper and Row, 1966), p. 119.

[3]Ibid., p. 124.

[4]Edward L. Meyen, "Guide Line for the Development of Life Experience Unit," University of Iowa Project PI 80 104, Special Education Curriculum Development Center, p. 2.

educable mentally retarded as well as the regular students. He felt that education for the individual child regardless of ability was basic. When making educational goals, he stated that we should keep in mind such principles as "the higher the correlation between what is taught and its application in life, the more successful the educational system."[1]

To have been aware of executing a planned curriculum and evaluating it was important and often neglected according to Meyen. He observed several classes and found a difference between the expressed goals and objectives for teaching and the planned curriculum objectives.

Trump and Baynham stated that educational professionals needed "constantly to seek new ways to organize classes, new methods of instruction and new ways of utilizing staff resources so that they will better serve the needs of youth and American life."[2] They advocated that the curriculum be divided into stages or steps. The individual's readiness to move was determined by "professional decision not by a test, a grade, or a unit of credit."[3] The rate of progress for each individual depended on previous achievement and his capacity to take the next step.

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[1] Ibid.

[2] J. Lloyd Trump and Dewy Baynham, Guide to Better Schools: Focus on Change (Chicago, Illinois: Rand McNally and Co., 1961), p. 217.

[3] Ibid., p. 57.

Lindley J. Stiles continued on the same approach, but referred to the necessity of having a creative teacher to utilize a curriculum. Stiles referred to a creative teacher as one who had a reliable knowledge of how students learn, how they react under a variety of stimuli and how learning becomes permanent and useful to the individual.[1]

Frank Riessman felt that before developing a curriculum we need to consider the topic of culture. He felt that educators needed a solid knowledge of their culture before they would know which techniques to use in gaining classroom rapport. Educators needed to extend the knowledge of the culture, to utilize it in their teaching even though the culture is not a similar culture to that of the educators.[2]

Snyder stressed that the working form of curriculum was important for the mentally retarded and that the broad concepts of education applied to all children with special ramifications for the mentally retarded child.[3]

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[1] Lindley J. Stiles, "Creative Teaching for Excellence in Education," School and Society, (September 23, 1959), 355-356.

[2] Frank Reissman, The Culturally Deprived Child (New York: Harper and Row, Publ., 1962), pp. 112-113.

[3] Elkan Snyder, "Learning Problems, Program Planning and Curriculum for the Mentally Retarded Child," The Special Child in Century 21, ed., Jerome Hellmuth (Seattle, Washington: Special Child Publication, 1962), p. 226.

In curriculum development the self-perception of a child coupled with their perception of the environment was important to understand, because these factors affect the interaction of a learner with his environment. Experiences differ among students and often are very different from our own experiences. How a child sees himself as well as how he sees his environment affects his learning progress.[1]

The mentally retarded child often resists change, learns at a slower rate, and has a weak ability to recall. He therefore needs special consideration in curriculum planning.[2] Snyder discussed the need for curriculum to be designed for purposive behavior. He felt that in developing curriculum goals that would develop the maximum potential of the individual as he functions in society.[3]

Kephart and Strauss stated that the influence of the environment varies according to the type of child. They refer to McCandless's statement that there is an interaction between "heredity and environment...nature and nurture."[4] Many things affect an individual's growth and some children

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[1] Ibid.

[2] Ibid., p. 227.

[3] Ibid., p. 232.

[4] Harvey A. Stevens, and Rick Heber (eds.), Mental Retardation (Chicago, Illinois: University of Chicago Press, 1964), p. 183.

are more sensitive to environmental influence than others. They felt that the heredity factor influenced a child's sensitivity to his environment.[1]

Frances Connor and Mabel E. Talbot conducted a research study to determine amounts and kinds of learning within the classroom. The research dealt with curriculum and teaching methods for the preschool educable mentally retarded child. The study indicated that many things affected an individual's growth and to varying degrees.[2]

Connor and Talbot caution that to know how children learn is not sufficient to ensure an effective learning process: children respond to teaching approaches differently. The research showed that curriculum presented in the "maintenance of conceptual wholes while directing attention to components of the whole," was effective.[3]

Professor Inhelder of Geneva, Switzerland, found a similar approach to curriculum and stated that reasoning "rests on the principle of the invariance of quantities: That the whole remains, whatever may be the arrangement of its

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[1]Ibid., p. 180.

[2]Frances P. Connor and Mable E. Talbot, An Experimental Curriculum for Young Mentally Retarded Children (New York: Teachers College, Columbia University, 1964), p. 300.

[3]Ibid.

parts, the change of its form, or its displacement in space or time."[1]

Bruner stated we needed to consider the individual child's development in curriculum to "challenge the superior student while not destroying the confidence and will-to-learn of those who are less fortunate."[2] He felt that we needed to develop curriculum along the sequence of psychological development because it is more closely related to the axiomatic order of a subject matter, than the historical order of development concepts within the field.[3]

Bruner stated that curriculum was built around the great issues, principles and values that a society deems worthy of the continual concern of its members. A diversified curriculum for a unique individual along developmental stages was established by three almost simultaneous processes which were involved in learning subject matter. Acquisition of new information (or refinement of previous knowledge), transformation (the manipulating of knowledge to make it fit a new task), and evaluation (checking whether the way we have manipulated information is adequate to the task).[4]

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[1]Jerome S. Bruner, The Process of Education (New York: Random House, 1960), p. 41.

[2]Ibid., p. 70.

[3]Ibid., p. 43.

[4]Ibid., pp. 48-52.

The three processes were utilized whether brief or long, with many or few idea episodes. Instructors were able to tailor materials to the individual student's needs and capacities by manipulating learning episodes.[1]

In developing the social needs of the brain injured child, Cruichshank described a technique that he referred to as the life-space interview. This interview was to build self understanding by an individual for his actions that were considered deviant. Describing and discussing his actions caused the individual to better understand himself, a necessity for improving one's self. This interview technique helped students whose deviant behavior interferred with their learning processes.[2]

Dr. Marie Egg also felt it important to develop the social needs of children. She was in accordance with educating a child in relation to the child stage of development and present situation. She stressed that in teaching it was important to consider the stimuli of daily life, and how it interacts on an individual's development stage and personality.[3]

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[1] Ibid., p. 49.

[2] William M. Cruichshank, The Brain Injured Child in Home, School, and Community (New York: Syrocuse University Press, 1966), pp. 266-267.

[3] Dr. Marie Egg, Educating the Child Who is Different (New York: The John Day Co., 1968), p. 79.

Even the child who was different was trained to employ his modest powers and use his existing abilities, and that "curriculum should contain knowledge for activities and other knowledge that is not limited to being practical so that his development as a human will not be impoverished."[1] Many factors are important to the development of children and affect their learning patterns, therefore children are unique, developing humans.[2]

To diagnose an individual child's learning problems an instructor should seek professional help when the classroom methods available are not adequate, for society has the advantage now of more precise testing procedures to help professional diagnose the learning problems related to children.[3]

After one hour of testing, a trained psychologist using the Stanford Revision of the Binet Simons Test can tell more accurately the difference in degrees of intelligence, superior to dull, than teachers by months of observations. Buck felt that psychologists should be used when there was a need to determine a degree of intelligence.[4]

To be sure that an individual's uniqueness was maintained professionals in the field of child development have

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[1]*Ibid.*, pp. 141-142. [2]*Ibid.*, p. 192.

[3]Buck and Zarfoss, *op. cit.*, p. 44.

[4]*Ibid.*, p. 32.

not generalized about anyone except for the purposes of research or statistics. They have dealt with the individual as being unique in all areas of development. Many types of tests have been developed to diagnose an individual child's development and show his uniqueness.[1]

It was necessary in developing an understanding of the individual child to review a few of the many tests and theories available in diagnosing an individual and his relationship to learning.

To measure the individual's ability to care for himself a test by Edgar A. Doll was developed called the Vineland Social Maturity Scale. After testing and the results were expressed, an index of relative growth was formulated.[2]

In describing a method to develop curriculum Hellmuth has written about Dr. Montessori's Theory. The theory was that at a certain stage of growth the child is in a sensitive period for the development of certain functions. One of the most important sensitive periods was the development of the five senses.[3] The materials Montessori designed were for use by the child independently and the principles of teaching the

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[1] Ibid., p. 67. [2] Ibid., p. 41.

[3] Lynn Mink, "Adaptation of the Montessori Method in Developing Visual Perception in the Special Child," The Special Child in Century 21, ed., Jerome Hellmuth (Seattle, Washington: Special Child Publication, 1964), p. 325.

individual child applied. "One of the primary purposes of the sesorial apparatus is to aid the child in organizing and classifying his impressions of the environment."[1]

The evaluation of the Montessori system was that it fills the need of aiding in sensory development of a child, but was to be a part of a richer and more varied curriculum.[2]

Woodward in her article on Piaget's Theory stated that this theory dealt with the process of intellectual development. It dealt also with evolution of abstract thinking through developmental stages of behavior of the individual. Piaget dealt in four main development stages: (1) sensorimotor, (2) preoperational stages (preconceptual, intuitive), (3) concrete operations, and (4) abstract questions.[3]

Different types of thinking are distinct categories. Through Piaget's theory new facts about the cognitive processes of mental defective thought have been brought about. According to Piaget individuals were placed in respect to their intellectual development in a rank order, and enabled a satisfactory differentiation of intellectual status to be made among

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[1]*Ibid.*, p. 326. [2]*Ibid.*, p. 340.

[3]Mary Woodward, "The Application of Piaget's Theory to Research in Mental Deficiency, *Handbook of Mental Deficiency Psychological Theory and Research*, Norman R. Ellis, ed. (New York: McGraw-Hill, 1963), p. 297.

individuals who would be outside the limits within which accurate differentiation can be made in terms of standard deviation units.[1]

In discussing the learning development of individual students Mayer referred to Jean Piaget's "central thesis that a child's thoughts before the age of eight is egocentric and syncrelistic...up to the age of eleven or twelve children are incapable of understanding relations as distinct from fact."[2] A child's previous training influenced his achievement on Piaget's tasks, so there was excellent reason to believe that he has substantially underestimated the capacities of children. Piaget asserted that his statements reflected the abilities of Geneva children of a certain social level at a certain point in time.[3]

Piaget's work has had a profound influence on intelligence tests in his attempt to discover how and why children succeed or fail on the sort of problems they got on intelligence tests.[4]

According to Gloria F. Wolinsky, Piaget's concepts were the idea of "perceptions different from intelligence, and that

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[1] Ibid., pp. 318-321.

[2] Martin Mayer, The Schools (New York: Doubleday, 1963), p. 105.

[3] Ibid., p. 107. [4] Ibid., p. 106.

the law of centrations has a relationship to perception."[1] Piaget's theory of perception has led to the examination of the assistive aspects that an environment can afford to its children as they proceed through life.

This theory by Piaget had the research workers seeking the sky, the teacher looking at any maladjustive thought processes and provided help in terms of an impairment of developmental functions; and the child ultimately must come to terms with an environment that provides and involves thought and meaningful action. With Piaget's theory perceptual structures also showed a change, that from the adaptive differentiations and combihative assimilations came the product of progressive construction.[2]

McCarthy and Kirk in 1961 presented a test, the Illinois Test of Psycholinguistics Abilities. This test was an approach to differential diagnosis to extend beyond classification of the Binet or Wechsler type test into an assessment which will suggest the area needing remediation.[3]

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[1]Gloria F. Wolinsky, "Piaget's Theory of Perception: Insight for Educational Practices with Children Who Have Perceptual Difficulties," Educating Children With Learning Disabilities, Edward C. Frierson and Walter B. Barbe, eds. (New York: Meredith Publ., 1967), p. 423.

[2]Ibid., pp. 426-433.

[3]Samuel A. Kirk and James J. McCarthy, "The Illinois Test of Psycholinguistic Abilities--An Approach to Differential Diagnosis," Educating Children With Learning Disabilities, Edward C. Frierson and Walter B. Barbe, eds. (New York: Meredith, 1967), p. 201.

The tests were constructed to "differentiate defects in (a) the three processes of communication, (b) the levels of language organization, and/or (c) the channels of language input and output."[1]

The test used as a diagnostic instrument led clues for remediation of deficits in various psycholinguistic functions found particularly among cerebral palsied, brain injured, and some emotionally disturbed children. A limitation of this test was that it did not make any assumptions with respect to neurological or neurophysiological correlates of behavior.[2]

It has been given in the related literature that it is important to consider the student as an individual, and adapt the learning situation to fit his needs. That there are many methods, theories, and tests to diagnose an individual's needs.

I. SUMMARY

According to Kephart and Strauss the environment in which a student lives affects his learning. Snyder reminds us of the importance of the self concept and its interaction to the environment in affecting the way in which a student learns.

Conner and Talbot included not only the environment, but the variables with the developmental stages that affect how a student responds to teaching approaches.

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[1] Ibid., p. 206. [2] Ibid., p. 216.

In helping to develop methods to teach the individual Hellmuth spoke of Dr. Montessori's Theory of developing the five senses. Wolinsky and Woodward discussed Piaget's Theory of intellectual development stages of the thought process. Connor and Talbot agreed with Dr. Inhelder in teaching conceptual wholes while not forgetting its component parts.

The curriculum goals that were considered in the related literature were taken into consideration when the model mathematics program was developed. Keppel felt it was important to link the variables of the curriculum together into a system of change. The linking of variables can be done with the model mathematics program. Differing cultures among the various buildings have been taken into consideration by the model program. This aspect of the curriculum was stressed by Riessman. Trump and Baynham asserted that the readiness stages within a curriculum should be made by professional decision, and not by testing alone. The readiness stages of the model mathematics program were determined by the instructor's professional decision. No matter what the curriculum, Stiles stressed, it took a creative teacher to utilize a curriculum. The model mathematics program and arithmetic guide was written for the creative teacher.

The model mathematics program for the academically deprived child does consider the individual and allows for a variety of diagnostic techniques to determine the needs of the individual. The model program is flexible in that the learning situations are adjusted to the individuals needs by the instructor.

In this chapter on the related literature the bases for the procedures and the development of a mathematics program for the academically deprived child were presented. The following chapters will describe the program in mathematics that was correlated with the findings stated in the related literature.

CHAPTER III

METHODS AND PRESENTATION OF DATA

In this chapter the fundamental procedures of the model mathematics program for the academically deprived child are described. Presentation of the data on the pilot study done in the area of the model program, and the basic designs of the program are given.

I. PROCEDURES

The traditional practice of giving more drill to an entire class when students miss problems has been widely accepted in elementary school classrooms throughout the nation. In the area of mathematics elementary instructors teach in the format of total group instruction and assignments. Representing a sincere effort to individualize mathematics instruction teachers have divided the classroom into groups for teaching similar to the groupings used in teaching of reading. The assignments and drill are then made on the group level. This practice has been critically appraised, and from this appraisal came the mathematics program for the academically deprived child.

The purpose of this field study was to develop a program which: (1) provided a model mathematics program for the academically deprived child (2) within the existing curriculum.

In addition to functioning within the existing curriculum the model program was designed to adapt to a changing curriculum. Too frequently programs have been designed that were not flexible enough to meet the changing needs of a school curriculum.

A three-fold purpose can be developed from this program:

1. the teacher rating form could be adapted to other subject areas
2. the open scheduling can be used for other curriculum areas
3. the basic design of the model mathematics program can be adapted to other curriculum areas.

II. IMPLEMENTATION

This model program utilized the teacher's skills in the area of individualized instruction. As the primary tool for placing and evaluating students in the model program a teacher rating form was developed. To schedule the program within the existing curriculum an open schedule form was used. Studies showed that it is important to teach the individual child, and that the quality of any program is dependent upon the quality of teaching. The model program is based on quality instruction of the individual child.

A pilot study that involved two teachers and fifty-seven students has been completed. One teacher did the instructing and the other teacher referred students to participate in the model program. The pilot study indicated:

1. that the model mathematics program is functional
2. that the program functioned within the existing curriculum
3. that the program did not require a schedule change
4. that the program used referrals without loss of regular academic curriculum time
5. that there was no additional cost for the program.

This was an exploratory study that placed a model mathematics program for the academically deprived child in the Webster City Community School System. This project served as a beginning for educators that kept abreast with new techniques and innovations.

Furthermore, this project exposed teachers to the utilization of valuable evaluation skills and heightened their awareness of being able to teach the individual child within the existing curriculum and schedule.

The teacher rating form was based upon the teacher's ability to identify and evaluate the educational and developmental needs of the individual child. The teacher rating form has four categories: (1) information, (2) analysis of area of need, (3) analysis of meeting the need, and (4) analysis of change.

TEACHER RATING FORM

TEACHER_________ GRADE______ BLDG.___________ DATE_________

STUDENT_________ I.Q.___ ARITH. TEST_____ SCORE_____ DATE____

INDIVIDUAL TEST SCORE_________ TEST___________ DATE_________

COMMENTS:

AREA OF WORK NEEDED:

GOAL DESIRED:

WORK DONE:

CHANGE NOTED:

CHANGE NOTED IN BASIC LESSON: NONE_____ SOME_____ GREATLY_____

		DATE	DATE
CHANGE AFTER ONE WEEK OR MORE:	REMAIN THE SAME	_____	_____
	GAINED	_____	_____
	LOSS	_____	_____

COMMENTS:

III. MODEL PROGRAM

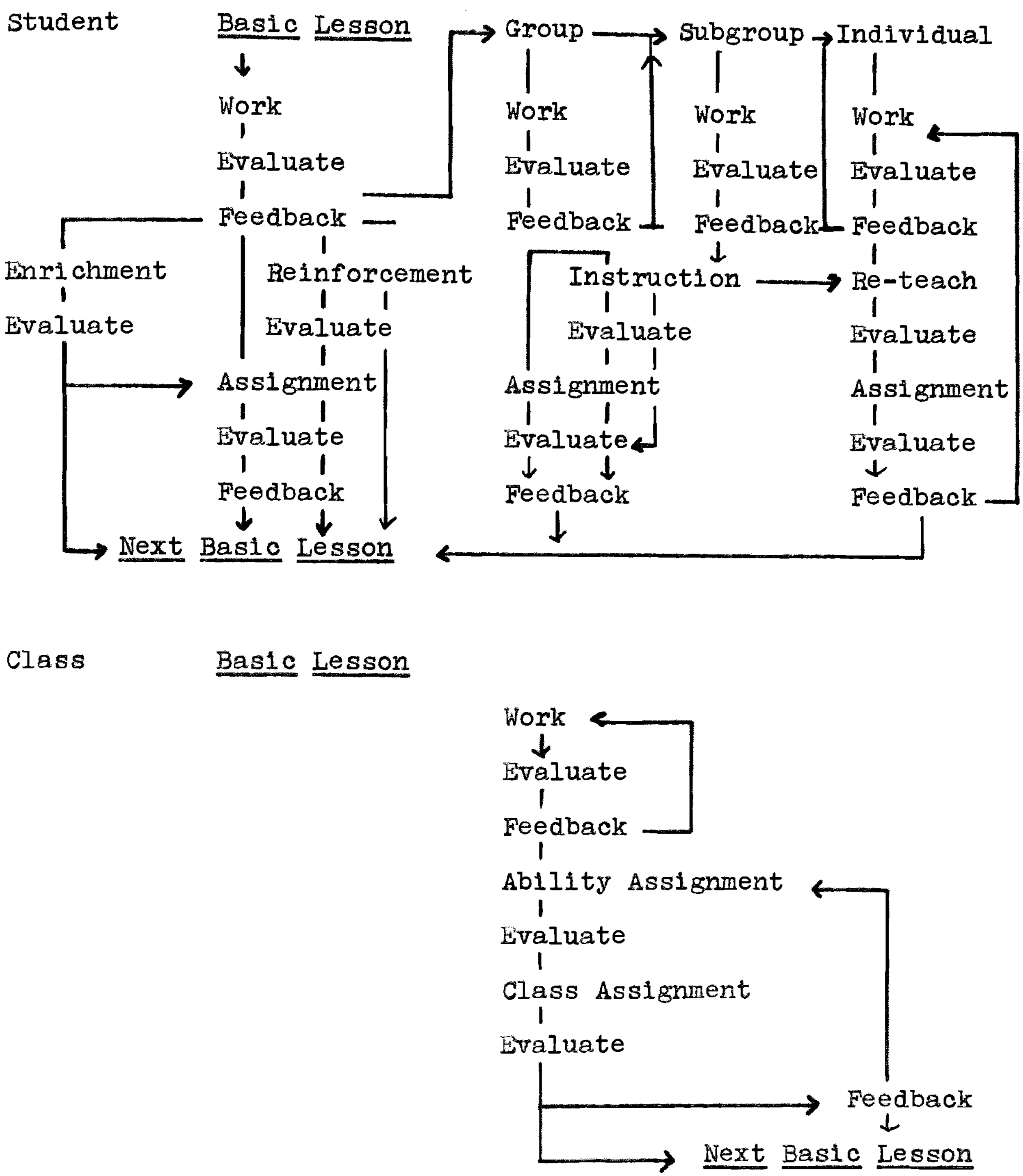
The basic lesson was presented. All of the children who had mastered the lesson advanced into more challenging material within the framework of the basic lesson. Those pupils who had experienced difficulty with some portion of the lesson were grouped according to the nature of the difficulty. The individual developmental stage problem was determined, and individual behavior goals set. If the student did not have an area of difficulty, then the student was moved into the assignment area. If the student did not need the assignment, then this student was given enrichment work. The teaching and work done was then in the format of individual behavior patterns and goals. Thus, the post-lesson grouping and small grouping were transitory, serving a specific purpose and dissolved upon achievement of that purpose. When the individual behavior goals were reached, all of the pupils would be together again in large group instruction and begin work on their next basic lesson.

It helped all students to progress steadily by teaching sequential basic lessons systematically to the entire group. The basic lesson was without the usual accompanying lock-step method, which decelerates the more capable student and increased the accumulation of difficulties for the less capable student.

The following design model shows the various stages through which the student proceeded according to the individual needs that were determined by a continuous evaluation.

The Table of Change design shows that many things were considered as input (what was put into the program), process (what actually was done in the program), and output (what was expected from the program). The Table of Change design has three main categories: (1) program, (2) lesson, and (3) technique.

MODEL PROGRAM



Feedback: Student confidence and attitude level in understandings

The evaluation design model shows that knowledge and skills can be considered as either input or output, as well as the model mathematics program itself.

The teacher rating form was used for evaluation as well as the lesson plan and evaluation design models.

To work with the existing schedule for a particular building in the pilot study it was necessary to schedule around the schedule for the special teachers (music and physical education). The open schedule design shows the type of schedule design that was used to adapt a schedule that coincided with the special teacher's schedule. By using an open schedule it allowed both the instructor of the model mathematics program and the referring instructor to adjust their daily curriculum and try various patterns before recommendations were made by these teachers for the first schedule.

The goal in this type of scheduling was that the instructor working with the referrals, and the instructor referring did not lose regular class time necessary for the regular curriculum.

TABLE OF CHANGE FOR THE MODEL MATHEMATICS PROGRAM

PROGRAM:

Input →	Process →	Output
Individual Ability	Model Program	Ability Change

LESSON:

Input →	Process →	Output
Individual Ability	Analysis Evaluation	Readiness Level for Program
Basic Lesson	Individual Ability Work	Data for Evaluation
Evaluation Results	Model Program	Individual Ability Work
Work in Model Program	Evaluation Individual Ability	Assignment
Assignment in Model Program	Evaluation Individual Ability	Evaluation of Assignment for Change
Change Evaluation Feedback (motivation)	Model Program	Readiness Level for Program

TECHNIQUE:

INTERACTION

Input →		Process →		Output
Involvement Ability Individual Evaluation Math Program	INTERACTION	Involvement Ability Individual Evaluation Math Program	INTER-ACTION	Involvement Ability Individual Evaluation Math Program

IV. EVALUATION

The model program was dependent upon a continuous on-going evaluation of:

1. General goals of the program.
2. Basic lesson goals.
3. Individual goals.
4. Work given to assess ability level, skills, knowledge.
5. Assignments in the area of learnings.
6. The how and the why of the student's progress.
7. What is being accomplished.
8. Type of change and its worth to the total individual goal.
9. Evaluation techniques.
10. Methods, techniques, materials.
11. Instructor.
12. Individual, feedback or self-evaluation and motivation.
13. The evaluation itself.

Each lesson in the model program is based on prior evaluation. The input into the lesson is based on evaluation, as well as the process and output of the lesson. The evaluation is based on <u>What</u> i want to know, <u>Why</u> I want to know it, and <u>How</u> do I know I know it.

MODEL PROGRAM

Input	$\longrightarrow$	Process	$\longrightarrow$	Output
Knowledge & Skills	Ability What & Why	Evaluation Interaction Individual	Ability How	Model Math Program

Input	$\longrightarrow$	Process	$\longrightarrow$	Output
Model Math Program	Ability What & Why	Evaluation Interaction Individual	Ability How	Knowledge & Skills

The teacher rating forms were designed to allow the teacher to record the information necessary for the type of evaluation and planning that was essential to the goals of the lesson being developed.

The lesson plan and evaluation forms were designed to record data that is essential to the individual teacher in accomplishing the goals of her program. The design can be utilized for the basic lesson, groups, individuals, or for all three for total evaluation as to the progress being done in the teacher selected goals.

The specific utilization of these forms are depended on the <u>creative teacher's</u> ability to: (1) evaluate the individual's learning processes, and (2) evaluate the developmental sequential steps of mathematics.

TEACHER RATING FORM

TEACHER______________ GRADE_____ BLDG._______ DATE________

STUDENT______________ I.Q._____ ARITH TEST_______________

SCORE___________ DATE_____________

INDIVIDUAL TEST SCORE_________ TEST_________ DATE________

COMMENTS:

AREA OF WORK NEEDED:

GOAL DESIRED:

WORK DONE:

CHANGE NOTED:

CHANGE NOTED IN BASIC LESSON: NONE_____ SOME_____ GREATLY_____

CHANGE AFTER ONE WEEK OR MORE:

	DATE	DATE	DATE	DATE
REMAIN THE SAME				
GAINED				
LOSS				

COMMENTS:

LESSON PLAN and EVALUATION

Student or Class__________
Date____________________

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
BASIC LESSON Goal: Work: Accomplished:				
GROUPS Goal: Work: Accomplished: Change:				
INDIVIDUAL Goal: Work: Accomplished: Change:				
EVALUATE				

LESSON PLAN and EVALUATION

Student ____________
Class ____________
Date ____________

BASIC LESSON	GROUPS	INDIVIDUAL	INDIVIDUAL	EVALUATION
GOALS				
WORK				
ACCOMPLISHED				
EVALUATE STUDENT				

PLAN SHEET

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY

SCHEDULE

ROOM #8

Monday & Wednesday		Tuesday	
9:50 - 10:20	MUSIC		
		10:25 - 10:50	P.E. Boys
10:30 - 10:45	RECESS	10:30 - 10:45	RECESS
		10:50 - 11:15	P.E. Girls
11:55 - 12:45	NOON	11:55 - 12:45	NOON
2:55 - 2:40	RECESS	2:25 - 2:40	RECESS

SCHEDULE ROOM #8

Thursday		Friday	
9:30 - 10:00	T.V.		
10:25 - 10:50	P.E. Girls		
10:30 - 10:45	RECESS	10:30 - 10:45	RECESS
10:50 - 11:15	P.E. Boys		
11:55 - 12:45	NOON	11:55 - 12:45	NOON
2:25 - 2:40	RECESS	2:25 - 2:40	RECESS

INSTRUCTOR'S AVAILABILITY SCHEDULE

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
(9:50 to 10:20)	with class 9:30 to 10:00 10:00 to 10:30	(9:50 to 10:20)	with class 9:30 to 10:00 10:00 to 10:30	
10:30 to 10:45 *	10:30 to 10:45 * (10:45 to 11:15)	10:30 to 10:45 *	10:30 to 10:45 * (10:45 to 11:15)	10:30 to 10:45 *
NOON	NOON	NOON	NOON	NOON
2:20 to 2:35 * **	2:20 to 2:35 * **	2:20 to 2:35 * **	2:20 to 2:35 * **	2:20 to 2:35 * **
3:30 to 4:00 #	3:30 to 4:00 #	3:30 to 4:00 #	3:30 to 4:00 #	3:30 to 4:00 #
Key: ( ) Planning time that could be used	*Recess when not on duty **adjustable study periods of class	# After School when not in conflict with meetings	Time now being used 2 hrs.	Maximum as schedule now is 6 hours

The factors of equipment, curriculum, teaching methods were kept flexible in the design of the model program so that they would be adaptable to any situation, as needed.

The teacher rating forms developed for the identification, work and evaluation of the students were so designed as to record whatever variables were needed in each individual case. This is in accordance with Keppel's belief that in planning and executing a curriculum that we cannot forget the many variables affecting the individual child.[1]

The behavior goals in the model program were stated by the individual instructor. The instructor has the opportunity to apply the necessary educational goals.

The evaluating of the curriculum utilized by the instructors is done on the individual case by a teacher rating form. From these forms a total picture can be given to see that the expressed goals and objectives are reached.

With the control of the learning under the direction of the instructor the cultural differences that might appear among the various buildings within a system can be accounted for and the techniques needed can vary according to the situation.

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[1]Frances Keppel, The Necessary Revolution in American Education (New York: Harper and Row, 1966), p. 119.

Taking into consideration the possibility that borderline or mentally retarded learners may be in the regular classroom the model program is in a working form of the curriculum. For learners who may have impaired functions the model program, being designed for the individual, can take the limitations of the various learners into consideration.

To carry out the principles upon which the model program was based it was necessary to have instructors both creative and able to make professional decisions in regard to the individual and the learning situation. This model program can be carried out by each instructor or it can be used by individual instructors referring students to one instructor, who then would teach and work with the individual students in reaching their behavioral goals.

The general teaching of arithmetic guide contains suggested guidelines for curriculum, teaching methods and equipment. From this guide the instructor developed an instructional program to help the academically deprived student.

The general teaching of arithmetic guide was first developed in 1963 while enrolled at Drake University in the Education Course 126 "Teaching of Arithmetic" instructed by Miss Bess Hamm. The fall of 1963 it was used as a guide to adjust to the new modern mathematics textbooks being used in our school system.

The guide was written in response to a need to find a working form of knowledge that could be used with a variety of textbooks and materials. Information was evaluated, condensed and put into reference form. One of the objectives was to bring forth what was a good arithmetic program, its functions, philosophies, theories, characteristics, text objectives, general objectives, teacher goals, and teaching techniques. Another objective was to generalize and put into working form how the elementary teacher could identify the individual needs of students in arithmetic, and develop guidelines for teacher awareness, and techniques.

The core of the guide was to simplify basic modern arithmetic methods in the areas of counting, readiness, addition, subtraction, base, multiplication, division, fractions, decimals, per cent and number line.

The activities, and equipment for teaching arithmetic were brief and mainly left open for ideas and placing of information as it became available and was of use to an individual teacher.

Meyen stated that there is a need for evaluation while executing a planned curriculum. The guide gives a teacher rating form and daily schedule form that can be used for continuing evaluation during the utilization of the model mathematics program.[1]

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[1]Meyen, op. cit., p. 2.

The section of the guide written about the individual and meeting needs was developed along the principles stated by Kingsley and Garry that an individual is unique and that an instructing professional has to be able to understand the relationships between what is needed and how the child learns.[1]

The guide is condensed and designed to be used by teachers as described by Lindley J. Stiles as a person with a reliable knowledge of how students learn, how they react under a variety of stimuli and how learning becomes permanent and useful to the individual.[2]

This chapter described the procedures for the model program in mathematics. The fundamental procedures described can be summarized in four steps: (1) basic lesson was presented, (2) groupings were made according to general areas of need, (3) individual developmental stages were determined, and (4) assignments were given according to the various needs.

The next chapter is the general teaching of arithmetic guide. In the guide is a teacher rating form that was developed to assist the instructor in keeping an on-going evaluation of the model program. This form was also used when instructors, not using the program, referred students

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[1] Kingsley and Garry, *op. cit.*, p. 67.

[2] Stiles, *op. cit.*, pp. 355-356.

to the instructor of this model mathematics program for instruction. The actual rating form was utilized as shown and then placed on 5 X 8 inch file cards for reference and evaluation.

Topics given in the guide are general and should be added to as the needs arise by the instructors of the program.

CHAPTER IV

TEACHING OF ARITHMETIC GUIDE

The guide was designed to give a brief example of the basic methods used in developing any mathematics program. The guide begins on the following page.

TEACHING OF ARITHMETIC GUIDE

CONTENTS

PREFACE

This guide was developed as a key to the basic principles that need to be applied to an individualized mathematics program. The guide is to be used as a reference from which a creative teacher can develop an individualized mathematics program.

In the first chapter will be found four main functions that are necessary before applying any of the techniques suggested in the guide. The teacher goals and techniques in this chapter are to be applied in the model program.

In the second chapter on the individual child, the teacher should have an awareness of the factors necessary to identify the individual child. In this section the teacher should apply the suggested techniques in teaching the individual child.

In chapter three a review of the basic operations in a mathematics program should be used as a reference from which a teacher can develop an instructional program.

The last chapters are suggestions of activities and equipment that can be used, and should be expanded by the addition of information that an individual teacher feels necessary.

CHAPTER I

A GOOD ARITHMETIC PROGRAM

FUNCTIONS: A good arithmetic program should have the following four functions:

1. Computational...This is the major function in many programs. Most programs over-emphasize this function of computation and drill. We should give more attention to understanding and meaning and keep a keen interest. Drill should be given only with a purposeful meaning of cementing the concepts and skills an individual child learns.

2. Informational...To enlarge upon topics and make them vital and real. We enrich and broaden the background of the students. Thus giving the student more meaningful experiences. To neglect the informational function in arithmetic is to fail to make arithmetic more interesting, and meaningful.

3. Sociological....Stress what pertains to the lives of the students. Thus a student will be more apt to put to use the skills he acquires in arithmetic. More emphasis should be placed on this sociological function than is now used in most schools.

4. Psychological...Must be trained numbers before developing quantitative thinking. Therefore numbers become a method of thinking. This is acquired through a period of experience with numbers.

PHILOSOPHIES: Three philosophies and theories of arithmetic are:

1. Authoritarian Philosophy...is the theory of <u>Drill</u>.

   Learn by memorization and habit rather than inquiry and discovery.

2. Laissez-faire Philosophy...is the theory of Incidental-learning.

   Has no set method or steps and treats arithmetic as learning only in when necessary situations arise.

3. Democratic Philosophy...is the theory of Meaning.

   Assumes the learner is capable to move from dependent to independent person and that he can make discoveries for himself. Then he can understand our number system.

CHARACTERISTICS: The common characteristics of any arithmetic program in an elementary school are:

1. Development of concepts.
2. Development of skills.
3. Development of oral expression.
4. Development of written expression.
5. Development of creative work.

ARITHMETIC TEXT: A good arithmetic text should provide stimulation and guidance of arithmetic development through meaningful situations, and should contain the following:

1. Have a program of readiness.
2. Have a program that is functional.
3. Have a program that includes inductive learning and quantitative thinking.
4. Have a program that has individual application.
5. Have a program that is flexible.
6. Have a program that includes an evaluation of the applications.
7. Have a program that is unified.

8. Have a program that is built on logical sequencial steps.

9. Have a program of inquiry and discovery.

OBJECTIVES: The general objectives of a modern arithmetic program should be:

1. Growth in ability to reason independently.

2. Growth in the power to discover and formulate meanings.

3. Growth in ability to solve problems.

4. Growth in ability to compute accurately and automatically.

5. Growth in ability to think quantitatively.

The arithmetic program in the elementary school should provide the arithmetic skills needed by the students such as:

1. Counting
2. Learn the decimal numeral system
3. Addition and Subtraction
4. Multiplication and Division
5. Fractional numbers
6. Concepts of measurement
7. Geometric ideas
8. Problem solving
9. Per cent
10. Graphs
11. Ratio and proportion
12. Sets and Sentences
13. Introduction to Algebra
14. Base

TEACHER'S GOALS: To strengthen any arithmetic program in the elementary school a teacher should:

1. Teach no longer at a group readiness rate, but at an individual readihess rate.
2. Teach the students the skills as they are ready for them.
3. Substitute personal conferences for over-marked arithmetic papers. Stress understanding of errors not do over attitude.
4. Make all the vicarious experiences as real, as visual, and as memorable as possible.
5. Review a skill in another situation in another way as soon as possible.
6. Always do many activities with objects before any recording is done, whether semi-concrete or concrete.
7. Don't count without counting something.
8. Always work for understanding rather than just the right answer.
9. Have the work to the student's level that he can do, but will still be a challenge to him, and be within his capabilities.
10. The only competition a student needs is to do the best he can in each step of arithmetic learning.
11. It is easier for a student to learn subtraction up to the 10 unit.
12. For students in the elementary grades it must be a physical activity before it can become a mental arithmetic problem or before he can record it.
13. Use lots of different objects so that students will arrive at a generalization without being told.
14. When adding a seen number to an invisible number it is hard for the children to visualize.

15. Have students be consistant in adding combinations, so that any drill work you make out the right combinations will be stressed and practiced.

TEACHING TECHNIQUES: Appropriate teaching techniques for stimulating arithmetic learning are:

1. Help a student to feel comfortable as he makes his oral contributions to the class.

2. Help a student to feel his contribution is important.

3. Give a student daily opportunities to express himself in meaningful oral activities.

4. Make sure the student feels he is an important member of the group.

5. Help the students to learn to interpret and to evaluate what they hear and see.

6. Have the students learn to interpret and to evaluate what is important to the learning situation and what is not.

7. Have students do their early work on subjects that they have a wide range of experience in.

8. Help students to discover standards for their arithmetic papers.

9. Create an environment in which a child learns that his ideas are respected.

10. Create an environment in which a child is motivated to do his best work according to his ability.

11. Create situations where a child may listen to distinguish between fact and opinion, to discover the main idea, to help organize his thinking, to make inferences and to arrive at judgments and different generalizations.

12. Show courtesy in listening to the students.

13. Provide interesting experiences which will challenge the attention of the learner.

14. Make lessons apply to actual situations.

15. Evaluate each child for individual improvement.

16. Through inductive teaching help the student to discover standards and rules that he will need to follow.

17. Use your text books as guides and reference books. But make sure you know your context of the text books and teach it in logical sequencial little steps.

CHAPTER II

THE INDIVIDUAL CHILD

TEACHER AWARENESS: A teacher's awareness of the students is important in identifying the needs of students. The teacher should:

1. Observe the child in class.
2. Watch for carry-over habits from home and previous grades.
3. Notice his work habits.
4. Teaching the child rather than material requires that attention to the individual need of the child must be of paramount interest.
5. The child can learn from mistakes.

INDIVIDUAL CHILD FACTORS: These factors need to be considered when identifying the needs of students:

1. Factors that affect children's learning are:

emotional	social	mental
mental age	interests	past experience
environment	health	Intelligence

2. Faith in himself is essential for the student.
3. Students vary greatly in the speed and ease with which they develop the skills of arithmetic.
4. In order to learn arithmetic a student must have the power to make associations. He must have the power to build his own pathways, to receive impressions and to respond to them. He must have the ability to see that associations can be made through visual experience. He must have the ability to hear so that he can receive and learn to comprehend the auditory stimuli that comes at him.

5. Personality by-products of learning are important for they remain with the students through life. These by-products are not always immediately apparent.

TECHNIQUES: Techniques that can be used to teach the individual child are:

1. Allow students to help in planning their own work.

2. Tests are to be used with students rather than on students.

3. Arithmetic learning should be co-ordinated with the increasing maturity and widening experience of the students.

4. Give the student the realization of achievement.

5. Encourage students to greater effort and success by making positive comments.

6. Begin with the student where he is and gradually take him as far as he can comfortably go.

7. Have the student learn from experience, inquiry and discovery rather than by memorizing a printed page.

8. Encourage a child to think things out.

9. Help the parents to become involved in their child's education.

10. Give purposeful homework.

11. Put T.Q.L.R. into effect (tune-in, question, listen, and review).

12. Always diagnose individual students, don't just correct his papers.

13. Help the student to learn from example rather than from direct teaching.

14. Help the student to acquire his meanings by both direct experience and by vicarious experience of reading and language.

15. Talk to him rather than at him.

16. How a child thinks is as important as what they think.

17. Supply the child's need when you find out what he needs.

18. Let the students build up their own meanings of number relationships and allow them to discover arithmetic.

19. Make sure the students have concrete experiences with quantitative thinking before you give them flash cards.

20. A student must have early meaningful experiences before he can build steps of meaning in numbers.

21. Mathematic competance adds to emotional stability.

22. Before a student can learn it must make sense to him. Therefore do not assume he learns, evaluate him to make sure he has learned the skill.

23. Use the principle of discovery, generalization and then testing of each student.

We arrive at these generalizations from his expressions and behavior fully as much as from his verbal response.

INDIVIDUAL

This design shows the developmental characteristics that should be taken into consideration when teaching the individual child.

Pre-School	Primary	Intermediate
PHYSICAL:		
Grows fast	Growth slow	Clumsy period
SOCIAL:		
Individualistic	Fond of people	Sex preference

PSYCHOLOGICAL:

<u>Mental</u>		
Experimental & Questional	Same	Wants reasons easily bored
<u>Emotional</u>		
High emotionality	Lessening emotionality	Comparatively stable
<u>Cultured</u>		
Seeks own pleasure	Seeks approval from all	Thinks about value, seeks reasons logical to him.

TEACHER RATING FORM

The teacher rating form used in the model program has four categories: (1) information, (2) analysis of area of need, (3) analysis of meeting the need, and (4) analysis of change. The categories of need, and meeting the need should show analysis of the developmental characteristics as well as the analysis of the work area of need.

CHAPTER III

MODERN ARITHMETIC METHODS

COUNTING: Counting is the bases of our arithmetic system.

1. Counting is the activity of evaluating a quantitative whole. Counting is not saying numbers in serial order. You can't count unless you count something.

2. Counting is the process of defining how many units make up a whole.

3. Before a child can count he must:

   a. Say numbers in order.

   b. Know objects in groups, discriminate parts of a whole.

   c. Know groups as whole...total.

4. We operate with numbers and write numerals.

5. In counting it is of an extreme importance that the students get real relationships as two is made up of one and one.

6. Ordinal numbers...is the order as first, second, and third. Cardinal numbers...are 1, 2, and 3 etc....how much, how many.

7. System of numerations is the way we write the name of the sequence of quantity or numbers.

8. Our system is based on the Hindu-Arabic origin. General principles are:

   a. Place Value

   b. Our system is additive

   c. Our system is base 10.

   (Base of a system of numbers is the number of units in any given place which must be taken to denote one in the next higher.)

9. Our system is based on ten digits.

READINESS: The arithmetic readiness program in the elementary grades should be based on the individual child. A teacher should:

1. Plan all activities around the student's needs, experiences, interests and math readiness.

2. Evaluate each student by observing him and testing him.

3. Know that students will not have much skill but will understand the following upon entering the readiness program.

    a. Basic number meaning.

    b. Basic vocabulary.

    c. Good attitude.

    d. Recognition of numbers.

    e. Interested in numbers.

    f. Confidence in handling number of objects.

ACTIVITIES: Activities for the readiness program are:

1. See how numbers function in their daily living.

2. Use number symbols.

3. Learn to use and understand the basic ideas of:

    a. Matching

    b. Counting

    c. Grouping

    d. Measuring

4. Help the students develop a method of attack upon simple problems.

5. Steps in making numbers make sense to children are:

    a. Object

    b. Picture

    c. Semi-abstract (visualization)

    d. Abstract symbol

ADDITION AND SUBTRACTION: Points to consider when teaching addition and subtraction are:

1. Help a child to be independent and to find things out for himself.

2. Use lots of different objects so the students can arrive at a generalization without being told the generalization.

3. Combining two or more groups of like objects is not adding. When you actually count the combined objects then you have adding.

    3 blocks in one hand
    2 blocks in the other hand
    5 blocks below the line tells me how many I have all together (total, amount, sum).

4. In combination always start with small numbers that are natural for the group.

5. Ask how many different combinations you can get? (factors)

6. Then move into subtraction of these combinations.

7. Do not use zero with combinations unless the need arises.

8. Three general steps in learning addition are:

    a. Discovery by putting together and taking apart groups of objects.

    b. Always build on from previous facts of learnings.

    c. Practice.

9. Always record what you actually did.

10. Subtraction in a physically sense is removing a part of a group from a larger group.

FACTORS:

Computative law of algebra...order of additives does not effect the sum.

2 + 3 = 5	a + b = c	. . + . . .	5
3 + 2 = 5	b + a = c	. . . + . .	5

How many different combinations of 5 can you make?

''' + ''	3 plus 2
'' + '''	2 plus 3
'''' + '	4 plus 1
' + ''''	1 plus 4

If I add 10's I get 10's.

''''''''''
'''''''''' is 2 10's 2 tens

If I add peaches I get peaches.

plus equals 7

REGROUPING:

Add this way as long as the students need to.

476		75	
289		17	
15	add 1's	12	add 1's
15	add 10's	8	add 10's
6	add 100's	92	
765			

Never half the numbers you record above the line and half the numbers you record below the line.

Regroup

$$\begin{array}{r} 476 \\ \underline{289} \\ 11 \\ \underline{655} \\ 765 \end{array}$$

SUBTRACTION: Three types of subtraction problems are:

1. Take Away or Less Than or Minus

   5 less 3 is ?

2. And What

   5 and ? is 7

3. The Difference

   The difference between 10 and 6 is ?

   How much more is 10 than 6 ?

METHODS:

Adding same amount to both minuend and subtrahend will not change the result.

$$\begin{array}{rl} 5 & \text{minuend} \\ \underline{-\ 2} & \text{subtrahend} \\ 3 & \end{array} \qquad \begin{array}{r} 5 \text{ plus } 2 \text{ is } \ \ 7 \\ \underline{-\ 2 \text{ plus } {-2} \text{ is } -\ 4} \\ 3 \end{array}$$

Semi-abstract decomposition subtraction. Use symbols for objects.

```
#####  ''         regroup  ####  ''  ''''''''''   record   52
 -##   '''''''              -##          '''''''          -27
                            ##    ''  '''                  25
```

Semi-abstract additive subtraction. Use symbols for objects.

```
#####  ''         regroup  ##### ''  ''''''''''   record   52
 -##   '''''''              -###         '''''''          -27
                           ##     ''  '''                  25
```

Additive subtraction.

$$\begin{array}{r} 3^{1}2^{1}4 \\ -\ 1\ 6\ 8 \\ \underline{1\ 1\ \ } \\ 1\ 5\ 6 \end{array}$$

8 from 14
7 from 12
2 from 3

NUMBER LINE

0 1 2 3 4 5 6 7 8 9 10

Move to the right to add.

Move to the left to subtract.

The number line is the graphic device for representing the sequence of numbers and relative size. Count the spaces or intervals not the points.

BASE

Base of a system of numbers is the number of units in any given place which must be taken to denote one in the next higher place.

Groups, sets, bundle, or packages:

'''''''''' '''''''''' is 20 or 2 groups of 10

20_{ten}

'''''''' '''''''' '''' is 2 groups of 8 plus 4

or

24_{eight}

Smaller the base the larger the recording.

45_{ten} is 1200_{three}

Base Ten	10 X 10 X 10	10 X 10	10	Ones
	1000	100	10	1's
	10^3	10^2	10^1	10^0
Base Three	3^3	3^2	3^1	3^0

BASE

46_{ten} equals $?_{six}$ Put 46 ones into packages of 6 and tell me how many packages you have.

36	6	1
1	1	4

$$\begin{array}{r} 46 \\ \underline{-36} \\ 10 \\ \underline{-6} \\ 4 \end{array}$$

1 package of 36, 1 package of 6 and 4 left over.

46_{ten} equals 114_{six}

114_{six} equals $?_{ten}$

How many 1's do I have X 4 equals 4
How many 6's do I have X 1 equals 6
How many 36's do I have X 1 equals $\underline{36}$
45

36	6	1
1	1	4

(1 X 36) plus (1 X 6) plus (4 X 1)

36 plus 6 plus 4 is 45_{ten}

114_{six} equals 46_{ten}

Same number of one's 114_{six} or 45_{ten}

''''''''''''''''''''''''''''''''''''
''''''
''''

base six

''''''''''   ''''''''''
''''''''''   ''''''''''
'''''

base ten

MULTIPLICATION: A short form of adding when groups are equal.

Points to consider when teaching the individual child are:

1. Student must know addition and what equal groups means before he can learn multiplying. (Early experiences will help a child to know groups but it will be quite a while before he knows what the word equal really means.)

2. It is important that the student know what we mean by multiplication.

3. Do not have the students learn the multiplication table at the expense of the multiplication facts.

4. It is easier for the students to learn multiplication if related to addition.

5. When you introduce multiplication always have the students use easy facts to start with.

6. Teach the new symbol X means time, could say add 5 three times.

Commutative Principle:

```
5 X 3 is 15            . . .        . . . . .
                       . . .        . . . . .
3 X 5 is 15            . . .        . . . . .
                       . . .
                       . . .
```

Semi-abstract:

```
##  ''                    ##  ''
##  ''    (22, 2 times    ##  ''
                        ----------
                        ####  ''''
```

Abstract:

```
 22
 X2
----
 44
```

Semi-abstract and Abstract:

```
       XX  ''                  22
           X2                  X2
     -----------             ------
####   ''''                     4    ''''
                                4    ####
                             ------
                               44
```

DIVISION: A series of subtraction of equal groups to find out how many. A teacher should:

1. Present general concepts and not something to memorize.
2. No process of division should be done without a keen analysis of what is being done.
3. The amount subtracted is the divisor, and shown as such.
4. Do not introduce ZERO until the need arises. ZERO is a symbol that represents nothing.
5. Long division is very hard for the students to understand so do not use long division until the students are ready.

Semi-abstract:

```
(555 ÷ 37)    ### '''''''  /CCCCC #####    '''''
                                  #####
                                  #####
                            -CCC #######              #
                            C#######       '''''
                           -C#######       '''''    '''''
                                                  # '''''
```

Abstract:

```
(55 ÷ 37)                  15
                      37/ 555
                          -37
                          185
                         -185
```

```
37/ 555  |
   -370  |  10   (10 X 37)
    185  |
   -185  |   5   ( 5 X 37)
            15
```

FRACTIONS: Points that a teacher should consider are:

1. Research has established that 90% of all fractions in use have the following denominators /2, /3, /4, /5, /6, /7, /10, /12, and /16.

   90% of the 90% used have denominators less than 10.

2. Do less pencil work and more reasoning and understanding. Teach the concepts or understandings of fractions.

3. Concrete experiences for fractions should come in the first grade because they are apart of their normal life.

4. First grade will acquire some knowledge without being taught.

   They do not understand that parts must be equal. They still think of the bigger half.

5. How you are going to name the number of parts to which the whole is divided is the name of the fraction.

   1/4   1/2

6. When the need arises let children discover:

   a. If the numerator is the same and the denominator gets larger the fraction gets smaller.

   b. If the numerator increases and the denominator stays the same the fraction gets larger.

7. Fractions represent number values.

   $2\overline{)1}$ another way   1/2   This is called unit factor. It means numerator is 1.

8. All division can be written as fractions.

   $23\overline{)645}$ 645/23

9. Two concepts of fractions   2/3   2 divided by 3

   a. 2/3 of 1 or 1 divided into three equal parts and you take two of them 1/3 plus 1/3 equals 2/3.

   b. 1/3 of 2 or take two 1's divided each into three parts. Take one of the three parts from each of the whole one's. 1/3 plus 1/3 equals 2/3.

FRACTIONS: The properties of fractions are:

1. Whole numbers are based on multiplication where fractions are based on process of division.

2. Fractions express comparison between two groups of objects.

3. Fractions express ratio.

4. Two kinds of fractions:

    a. Proper...numerator is less than the denominator.

    b. Improper...whose numerator is either the same or larger than the denominator.

5. Related means small denominator is contained in the larger denominator.

6. Unrelated means no common denominator.

7. If the fractions are not related divide the opposite denominator into its number.

8. In the problem 1/2 plus 1/4 ask: How many 1/4ths are there in 1/2?

METHODS:

Set the Social Situation! I have 1/4 piece of a pie and 1/2 of a piece of pie. How many pieces would I have if they were all fourths?

Manipulative Level! This is the first step in fractions. Give the students concrete material they can use to fold, cut and measure the parts of a circle or ruler.

ADDITION AND SUBTRACTION OF FRACTIONS:

1. Largest denominator, the common denominator. (Unlike but related) 2/1 plus 1/4 are how many fourth's? Change 1/2 to fourths, then add the fourths. We find that we have 3/4. Let us record what we did. 1/2 is equal to 2/4, we have 2/4 plus our 1/4 so we have 3/4.

2. No common denominator. (unrelated) To find out what I have let us put 1/2 circle on the whole, then 1/3 on the same whole. Take the size piece left and fold each piece, (1/2 and 1/3) to find out how many of that size piece is in each one.

   How many did I fold the 1/2 into? (3) How many did I fold the 1/3 into? (2)

   Let us put the 1/2 and 1/3 on the whole and find out how many more pieces I need to make a whole.

   If I had that piece how many pieces do I have? Let us record what we did. 1/2 equals 3/6 and 1/3 equals 2/6. I count the number of sixths. How many did I have? (5)

3. Mixed numbers. Add or subtract our wholes, add or subtract our fractions.

4. How to get a common denominator: How many parts do I divide my sixth's into? How many parts do I divide my fourth's into?

   1/4 plus 1/6

   What have I found? I have found the common denominator to be 24.

   We find that we can combine these in groups. The denominator is now 12 because I could combine the 2's. Ask this as questions so students will discover.

   Let us record what we have just done.

   1/4 has the denominator of 4. What two factors gives us 4?

   2 X 2

   1/6 has the denominator of 6. What two factors gives us 6?

   2 X 3

   1/4 equals 1/2 X 2 plus 1/6 equals 1/3 X 2

   I find the two in each denominator will use the two only once. I divide 1/4 into 3 parts, I divide 1/6 into 2 parts.

   1/4 into 3 parts equals 3/12
   1/6 into 2 parts equals 2/12

   I find that I have 5/12

MULTIPLICATION OF FRACTIONS:

How many times do I see it?

Multiplicand	What I have.
X Multiplier	X How many Times I See It.
Product	Product

I want to know how many times I see it not segiment size.

I have (I have all but 1/4 of a circle)

Divided it into three equal parts

3/4, once Fold one-third down

3/4, 2/3 of a time Fold another third down
(2/3 X 3/4)

3/4, 1/3 of a time (1/3 X 3/4)

or:

3/4, 1 time fold into halves

3/4, 1/2 of a time. (1/2 X 3/4)

Can multiply a unit fraction times a whole number or a unit fraction times a unit fraction.

Multiplicand	What you have.
Multiplier	How many times you repeat it.
Product	

DIVISION OF FRACTIONS:

Set a social situation! I have 1/4 of a circle here and I want to share it equally with John and Mary. How many pieces will I need? (2) What is another way of saying how we want to share this 1/4 of a circle I have? (divide it into 1/2)

Manipulative situation! What did you tell me I should do with this fourth of a circle? (divide it into 2 equal pieces or into 1/2) Fold a circle and demonstrate. Then record what we just did. Work it with semi-abstract symbols, then abstract symbols.

Three step method of division of fractions: 1/4 divided by 2/3. How many times can I subtract 2/3 from 1/4? None, so: 1/4 divided by 1 is 1/4 (any quantity divided by one is the quantity). 1/4 divided by 1/3 is times times as many. 1/4 divided by 2/3 is twice as large as 1/3 so would be one-half as many.

1/4 divided by 2/3 equals 1/4 X 3 divided by 2 equals 1/4 X 3/2 equals 3/8 of a time.

1. Find out how many one's in the dividend.
2. Find out how many 1/3 in the dividend. (3 times as many)
3. Find out how many 2/3 are in the dividend. (1/2 as many because 2/3 is twice as large, Since 1/2 as many so divide by 2)

Reciprocal method: Fraction or mixed number can use the inversion or reciprocal method. Replace the divisor by a whole number is the simplist form of dividing fractions. Reciprocal is any two numbers whose product is always one.

3/4 divided by 2/3 or $\dfrac{3/4}{2/3}$

Find the reciprocal $\dfrac{3/4}{2/3 \times 3/2 \text{ equals } 1}$

Must multiply the numerator by the same number you multiplied the denominator.

$$\frac{3/4 \times 3/2}{1} \text{ equals } \frac{9/8}{1} \text{ or } 1\ \ 1/8$$

DECIMALS: In 1585 a paper was written on the decimal system. Not until the 18th Century were decimals used. Then in the 19th Century President Jefferson was the first man to use a decimal system as a base for money.

1. A number system which uses ten as its base is a decimal system.

2. A decimal fraction is a fraction whose denominator is a power of ten when it is written in decimal form. The exponent is designated by the number of digits to the right of the decimal point.

3. The decimal value depends on its place value.

4. Integer is a numeral that notates a group of decimal units.

5. All notative numerals can be called decimals or decimal fractions.

6. Kinds of decimals:

   a. An integer is a decimal with a place value of 10, 100, etc.

b. A decimal fraction is a decimal number with a value less than one. It might be considered as a common fraction with a denominator of 10.

c. A mixed decimal number combination is an integer and a decimal fraction.

7. The decimal point is not a part of the decimal system. It is a punctuation mark (.) that is an adopted symbol or sign to show a need for change in our thoughts. It shows us where the one's column is.

8. Decimal fractions the denominator is expressed by its place value.

   .3 equals 3/10 .03 equals 3/100

9. The number of zeros in the denominator shows the number of places to the right of the one's column.

   .8 equals 8/10 .25 equals 25/100

10. Addition and subtraction of decimals add or subtract like quantities: tenths, with tenths, hundredths with hundredths.

11. When multiplying decimals to mark the decimal place in the product you add up the number of decimal places in the multiplicand and the multiplier. Count over from the last place value on the right in the product the same number of places and record your decimal point.

12. In the division of decimal fractions when a decimal fraction is divided by an integer the denominator is not changed.

13. When you divide an integer by a decimal fraction ask what do we want to know.

8 divided by .4	How many 4/10 there are in 8
8 X 10 equals 80/10	Have to know how many 1/10 in 8
$\frac{80/10}{4/10}$ equals 20	How many 4/10 are in 80/10

14. When you divide an integer by a mixed decimal fraction find out how many, in this case, tenths.

    6/1.5 — 6 X 10 will have 60 tenths.

    60 tenths, how many times can I subtract 15 tenths? (4 times)

    (multiply by 10 because we are finding out how many tenths are in the integer.)

PER CENT: Per cent is derived from the latin word per centum meaning by the hundred. The symbol that is used is derived by the 100, % two zeros and the one to divide the symbols.

1. Per cent is a common or decimal fraction whose denominator is 100.

   6% means 6 out of 100 or 6/100 or .06

2. Fractions and decimals can be expressed as a per cent.

CHAPTER IV

ACTIVITIES FOR TEACHING ARITHMETIC

1. Arithmetic serves everyday needs of the children.

   a. Keep a large chart to put various problems on, that the students desire to know the answer to.

   b. Keep a chart of the arithmetic activities the students like to do.

2. Arithmetic is necessary in every subject taught in school.

   a. Make a folder of new words and the meanings of these words that apply to arithmetic.

   b. Use newspaper advertisements to concoct original arithmetic problems.

   c. Have students bring in arithmetic problems relating to the various subjects and then have them give reports on their findings.

3. Arithmetic skills are learned best when practiced in meaningful situations.

   a. Have the students keep records of visitors, as to the number of visitors, time, and date of visit.

   b. When planning a field trip have the students note and work out the various mathematical problems that arise.

4. Arithmetic skills are best developed when provision is made for guided practice in new learning and maintenance review of previous learning.

   a. Keep the spirit of discovery and the interest that comes with inquiry when reviewing previous learnings by using new activities in your teaching.

   b. Do blackboard work to review and illustrate skills. As number is an idea. A numeral is a name for that idea. The words, figures, or symbols which we read, write, or erase are numerals.

c. If a student has difficulty in writing the numerals use the Kinesthetic experience method to help him form the figures.

5. Arithmetic instruction must take into consideration individual needs and abilities.

   a. Have the students conduct a survey of their own to determine how much mathematics is used in their daily life.

   b. Make practical applications of the mathematics being studied in activity problems.

   c. After a student has completed a skill help them to apply their knowledge and skill in new situations.

6. Arithmetic is necessary and significant for achievement of social competency.

   a. Have students use their arithmetic skills in class work, as the number of milk needed for rest time.

   b. Have students make personal address and phone books.

7. Arithmetic is enhanced through motivation toward creative expression.

   a. Have the students make up activities for the various skills learned.

   b. Have them keep a folder on how they use the skills they have acquired.

   c. Give them action pictures and have them describe all the mathematical ideas in the picture.

   d. Have the students dictate and write number sentences. Have them tell number stories, dramatize them and then summarize them in the form of number sentences. Action pictures or concrete objects can be used.

CHAPTER V

EQUIPMENT

VTR Equipment: The video tape recorder can be used to present the basic lesson.

It can be used for ability instruction for:

1. enrichment
2. reinforcement
3. instruction
4. reteaching

Listening Centers: The listening centers can be used to present the basic lesson.

It can be used for ability instruction for:

1. enrichment
2. reinforcement
3. instruction
4. reteaching

The use of the above equipment illustrates possible ways the instructor of the model program can present more than one basic lesson or ability instruction to more than one class, group or individual during one period of time.

The use of programmed materials and other audio-visual aids, as well as instructor-made materials is limited only by the ability of the teacher to utilize all available equipment.

The materials in the Webster City Community School District is available through the IMC center, and is sent to the teachers upon filling out a request form and sending it to the center through the interschool mail system or by placing a phone call to the IMC center.

CHAPTER VI

GLOSSARY

ASSOCIATIVE PRINCIPLE: The way of combining three or more objects two at a time is said to be associative if the result of the combination of the three or more objects (order unchanged) does not depend upon the way in which the objects are grouped.

ADDITION: When you count the combined objects of 2 or more groups of like objects.

BASE OF A SYSTEM OF NUMBERS: The base of a system of numbers is the number of units in any given place, which must be taken to denote 1 in the next higher place.

CARDINAL NUMBER: A cardinal number is a number that tells how many objects a set has.

COMMUTATIVE PRINCIPLE: A way of combining sets of objects two at a time is said to be commutative if the result of the combination of two objects is not affected by the order in which the sets of objects have the operation applied.

COMMUTATIVE LAW OF ALGEBRA: Order of additives does not effect the sum.

COUNTING: Is the activity of evaluating a quantitative whole.

DECIMAL FRACTION: A decimal fraction is a fraction whose denominator is a power of ten when it is written in decimal form. The exponent is designated by the number of digits to the right of the decimal point.

DECIMAL SYSTEM: A number system which uses ten as its base is a decimal system.

DISJOINT SET: No member of one set is a member of the other set.

DISJOINT SUBSET: No member of one subset is a member of the other subset.

DISTRIBUTIVE PRINCIPLE: The distributive principle of multiplication with respect to addition asserts that the product of the multiplier and the sum of two or more addends is equal to the sum of the products of the multiplier times the separate addends.

DIVISION: A series of subtractions of equal groups to find out how many.

EQUAL: Two expressions are said to be equal if they denote exactly the same quantity.

EQUATION: A special kind of number sentence, an expression of equality between two quantities, is said to be an equation.

EQUIVALENT SET: Sets that contain the same number of members.

EVEN NUMBER: An even number is an integer that is devisible by 2 with no remainder: a number which contains the factor 2 at least once.

EXPONENT: An exponent is a small symbol, usually a numeral, to the upper right of a base numeral, which tells us how many times the base numeral is to be taken as a factor.

FACTOR: A factor is any of two or more quantities which form a product when multiplied together. When we refer to the factors of a whole number, we usually mean a set of whole numbers whose product is the original whole number.

FRACTION: A fraction is an indicated quotient of two quantities.

INTEGER: An integer is a whole number.

MATRICES: A rectangular array of numerals lined up in rows and columns.

MULTIPLICATION: A short form of adding when groups are equal.

NUMBER: Number denotes the concept of quantity which starts with the cardinality of a set of objects.

NUMERALS: Numerals are symbols used to denote numbers.

ODD NUMBER: An odd number is a number which is divisible by 2 with a remainder of 1.

OPERATION: An operation is a specific process for combining two or more quantities.

ORDINAL NUMBER: An ordinal number is a number which indicates the position or order of a member of a set in relation to other members of the set.

PARTITIONING: Separating a set into two or more disjoint subsets or joining two or more disjoint subsets to form a set.

PATTERNS: Generalization to predict how numbers will behave under given conditions.

PER CENT (HUNDREDTHS): Per cent is a fraction having 100 as a denominator; a per cent is a number considered as a ratio to 100. It may be expressed as a fraction or a decimal.

PLACE VALUE: The term place value denotes the value assigned to a digit by virtue of its position in relation to the one's place.

RATIO: A ratio is the indicated quotient of two quantities compared by division; the relative size of two quantities.

SENTENCE (NUMBER SENTENCE): A number sentence is an expression which indicates a relationship between two or more numbers or quantities.

SET: A set is an aggregate, collection, group, family, etc., of particular things. A collection of things.

SETS, UNION OF: Union of sets denotes the set consisting of all elements belonging to at least one of the sets forming the union.

SUBTRACTION: In a physical sense is removing a part of a group from a larger group.

SUBSET: A part of a set.

SYMBOLS: Recordings that give meaning to number sentences.

UNION: Two sets joined to make a set of all numbers of the two sets.

APPENDIX

MEASUREMENTS: The key to working with measurements is to be sure of the understanding of what you are measuring and what you would be re-grouping.

2 yds.	2 ft.	9 inches.	
	x	4	
8 yds.	8 ft.	36 inches	then regroup according to:
			12 inches equals 1 foot 36 inches equals 1 yard 3 feet equals 1 yard
11 yds.	2 ft.		

GEOMETRY: When discovering work conclusions into mathematical sentences for understanding use known facts from previous understandings to give a base concept. Finding the hypotenuse of a right triangle.

a^2 plus b^2 equals c^2

3^2 plus 4^2 equals 5^2

9 plus 16 equals 25

When working with unknowns ask how you would work it from an early learned elementary unknown, then substitute the harder unknown and work the problem.

$4\frac{1}{2}$ divided by N is 18

Think: 6 divided by N is 3

Ask: How did I know the answer was 2?

What did I do?

Then work the problem the same way.

GRAPHS: Use as a visual form when working with fractions and per cent.

CHAPTER V

SUMMARY

The model mathematics program for the academically deprived child utilized the methods, theories and various methods of testing as found in the related literature.

Throughout the related literature the child or student was referred to as an individual and it was indicated that the individual is <u>unique</u>.

To be instrumental in aiding the individual's growth then one must be aware of the vast range of abilities,,and have reliable knowledge in the area of understanding the various development stages of growth of the individual in regard to how a child develops in learning and makes it permanent and useful. That the processes of development are simultaneously going on, the need to understand the relationship between the varying stages or steps of development is necessary.

The decisions in curriculum are influenced by the culture in which the curriculum is designed. An individual's readiness as determined by professional decision should determine where he is in the developing stages. The professional decision should take into account an individual's previous achievement and capacity for the next step.

The model program's goal was in making a working relationship for attaining a positive relationship between curriculum development and the individual child's ability. The model mathematics program was followed through so that the expressed goals and objectives were actually carried out, thus giving a positive working relationship between the factors of curriculum and individual ability.

I. CONCLUSIONS

The pilot study for the model mathematics program for the academically deprived child was evaluated by the administrator and teachers involved in the study. Based upon the decision that the pilot study appeared successfully completed the following conclusions were drawn:

1. That in the future the model program should be used in the building where the pilot study was done.
2. That after the model program has been used for a period of three years plans to extend the program to other buildings should be made.
3. That an in-service program be developed for all teachers involved in the model program.
4. That a continuous evaluation program should be written for the model mathematics program.

II. RECOMMENDATIONS

The following recommendations were made for the model mathematics program for the academically deprived child:

1. A continuous evaluation program should be developed for the model mathematics program.
2. The guide and model program should be adapted for all elementary buildings should the experimental program evaluation indicate an expansion was desirable.
3. Revisions of the model mathematics program should be made annually.
4. An extension of the model mathematics program should be developed for the summer school session.
5. The basic design of teaching the individual child, and the adjustment of materials to fit the needs of the learning should be considered for other subject areas.

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